Central European Journal of Economic Modelling and Econometrics

Adjustment Processes Resulting in Equilibrium in the Private Ownership Economy

Agnieszka Lipieta*

Submitted: 13.09.2017, Accepted: 10.10.2018

Abstract

The paper considers a private ownership economy in which economic agents could realize their aims at given prices, Walras Law is satisfied but agents' optimal plans of action do not lead to an equilibrium in the economy. It means that the market clearing condition is not satisfied for agents' optimal plans of action. In this context, the paper puts forward three specific adjustment processes resulting in equilibrium in a transformation of the initial economy. Specifically, it is shown, by the use of strict mathematical reasoning, that if there is no equilibrium in a private ownership economy at given prices, then, under some natural economic assumptions, after a mild evolution of the production sector, equilibrium at unchanged prices can be achieved.

Keywords: private ownership economy, equilibrium, Walras Law, adjustment processes

JEL Classification: D5, D2

*Cracow University of Economics; e-mail: alipieta@uek.krakow.pl

1 Introduction

Joseph Schumpeter was the first to identify innovations as the essential changes which could disturb equilibrium in the economy (Schumpeter 1912). His vision of economic evolution was strongly inspired by Walrasian thinking (see for instance Andersen, 2009; Hodgson, 1993; Shionoya, 2015) but instead of focusing on the properties of the equilibrium outcome, he viewed it only as a starting point for the study on economic development. Schumpeter also discussed two different mechanisms which can potentially ensure convergence to the equilibrium in the economic systems: the *tatonnement* mechanism, which results in a state of Walras equilibrium (see Walras, 1954; Uzawa, 1960; Joyce, 1984) as well as the creative destruction mechanism, which assumes coexistence of two opposite processes, namely introduction of new commodities, new technologies and new organizational structures etc. and elimination of the existing, outdated solutions, jointly leading the economic system to a new equilibrium state. In addition to these well-known mechanisms, the results obtained in Lipieta (2013, 2015a, 2015b) signify that convergence to an equilibrium in the economy can be obtained not only through the market mechanism but also through mechanisms designed and driven by a person or an institution arranged by law (see also Hurwicz, Reiter, 2006). In this research, such a person or an institution will be called the manager of the production sphere, shortly called the manager.

At a considered time interval, the process of setting of equilibrium can be also regarded as one of the stages of Schumpeterian economic evolution, even if an innovation does not appear at the given period on the market.

This paper proposes new, interpretable adjustment processes resulting in equilibrium in a private ownership economy (see Debreu, 1959; Arrow, Debreu, 1954; Mas-Colell et al., 1995), which corresponds to Schumpeter's thinking (see Shionoya, 2015; Lipieta, Malawski, 2016). The evolution of economic agents' activities, sufficient for the existence of equilibrium in an exchange economy with production in which, at given prices, the Walras Law is valid, is presented, differently from results obtained so far (compare to Radner, 1972; Magill, Quinzii, 2002). To consider time in a private ownership economy, which is the static economic model, the adjustment processes defined by L. Hurwicz (see for instance Hurwicz, 1987) are applied.

Adjustment processes have been already used in the theory of economics. Classical theories concerned, above all, the determination of equilibrium prices or properties of equilibrium (such as stability and uniqueness). The results can be found, for example, in Arrow, Hurwicz (1958, 1959), Hurwicz (1987). The analysis on the adjustment processes in which producers play an active role can be found in Lipieta (2015a, 2015b), where the trajectories leading to equilibrium in the economy with linear consumption sets (see for instance Moore, 2007) were determined.

An adjustment process is a mathematical structure by the use of which economic processes at discrete time can be modelled. The domain of an adjustment process is the so called set of environments – the set of characteristics of economic agents. An adjustment process is the triple consisting of a message space, a response function and an outcome function. A response function assigns to messages, sent at given time on markets by economic agents, messages sent in the next point of time. These messages result from analysis of the information available at earlier point of time. An outcome function assigns to messages, goals of economic processes. The rules of an outcome function can be determined by the market mechanism or can be constituted by a manager. In this research, the economic environments form a private ownership economy, while messages – plans of action of economic agents at given prices.

This paper proposes three new, economically interpretable adjustment processes which result in equilibrium in a private ownership economy. Specifically, under the assumption that Walras Law is satisfied at given prices, some processes with discrete time, which transform the production sphere of a private ownership economy are modelled in Theorems 1-4. These processes transform a sequence of given producers' and consumers' optimal plans of action from the initial economy into a state of equilibrium in the economy with modified product sector in such a way, that consumers' plans of action and consequently their optimal plans are not changed. The processes under study can be viewed as dynamic processes leading to equilibrium in the static model while Theorems 1-4 can be seen as the alternative versions of the Arrow and Debreu theorems on existence of equilibrium in a private ownership economy (see Arrow, Debreu, 1954; Mas-Colell et al., 1995; Maćkowiak, 2010). Let us emphasize that the assumptions used in the current paper are other than those considered by Arrow and Debreu in Arrow, Debreu (1954).

The paper consists of five sections. In the next part, the private ownership economy is defined. In the three section, the definition of adjustment processes is presented. The fourth section is devoted to modelling adjustment processes resulting in equilibrium in the analysed economy. The last part lists conclusions.

2 The private ownership economy

In this part of the paper, we shortly demonstrate the construction of the private ownership economy and the Debreu economy (see also Lipieta, 2013) defined in the form of the multi-range relational system (see Adamowicz, Zbierski, 1997).

Consider linear space $\mathbb{R}^\ell$ ($\ell \in \{1, 2, \dots\}$) with standard scalar product

$$(x \circ y) = (x_1, \dots, x_\ell) \circ (y_1, \dots, y_\ell) = \sum_{k=1}^{\ell} x_k \cdot y_k,$$

as the commodity – price space. The activities of two groups of economic agents: producers and consumers are under our consideration. The aim of producers is profit maximization, the consumers' aim is to maximize preferences on budget sets. Let

$B = \{b_1, b_2, \dots, b_n\}$ be the set of producers, $n \in \mathbb{N}$.

$\delta: \; B \ni b \to Y^b \subset \mathbb{R}^\ell$ be a correspondence of production sets,

$p \in \mathbb{R}^\ell$ be a price vector.

Every vector $y^b \in Y^b$, called the plan of producer b, is identified with the production process of this producer, with outputs and inputs given by vector y^b.

Definition 1. *A two-range relational system*

$$P_q = (B, \mathbb{R}^\ell; \delta, p)$$

is called the quasi-production system. If for given price vector $p \in \mathbb{R}^\ell$

$$\forall b \in B \quad \eta^b(p) \overset{\text{def}}{=} \{y^{b*} \in Y^j : p \circ y^{b*} = \max\{p \circ y^b : \ y^b \in Y^b\}\} \neq \emptyset,$$

then quasi-production system P_q *is called the production system.*

The elements of set $\eta^b(p)$ will be called the optimal plans of producer b. Let us emphasize that in quasi-production systems, the aim of producers is not determined, in contrast to the production systems, where producers aim in profits maximization. Similarly, let

$A = \{a_1, a_2, \ldots, a_m\}$ be the set of consumers, $m \in \mathbb{N}$,

$\Xi \subset \mathbb{R}^\ell \times \mathbb{R}^\ell$ be the family of all preference relations,

$\chi : A \ni a \to X^a \subset \mathbb{R}^\ell$ be a correspondence of consumptions sets,

$\epsilon : A \ni a \to \omega^a \in X^a$ be an initial endowment mapping,

$\varepsilon \subset A \times (\mathbb{R}^\ell \times \mathbb{R}^\ell)$ be a correspondence, which to every consumer $a \in A$ assigns a preference relation $\preccurlyeq^a$ from set Ξ restricted to the consumption set X^a,

$p \in \mathbb{R}^\ell$ - a price vector.

Similarly, every vector $x^a \in X^a$ the plan of consumer a, is identified with a consumption process of this producer, with outputs and inputs given by vector x^a.

Definition 2. *The three-range relational system*

$$C_q = (A, \mathbb{R}^\ell, \Xi; \chi, \epsilon, \varepsilon, p)$$

is called the quasi-consumption system. If, for given price vector $p \in \mathbb{R}^\ell$*,*

$$\forall a \in A \quad \beta^a(p) \overset{\text{def}}{=} \{x \in X^a : p \circ x \leq p \circ \omega^a\} \neq \emptyset$$

$$\forall a \in A \ \varphi^a(p) \overset{\text{def}}{=} \{x^{a*} \in \beta^a(p) : \quad \forall x^a \in \ \beta^a(p) \quad x^a \preccurlyeq^a x^{a*}\} \neq \emptyset,$$

then quasi-consumption system C_q *is called the consumption system.*

The elements of set $\varphi^a(p)$ are called the optimal plans of consumer a. In both quasi-consumption systems as well as in consumption systems, consumers aim at maximizing their preferences on budget sets, but in quasi-consumption systems there may be no upper bound for a preference relation on the given sets.
Let $p \in \mathbb{R}^\ell$ be a price vector, P_q – a quasi-production system and C_q - a quasi-consumption system in space $\mathbb{R}^\ell$. It is assumed that every consumer shares in producers' profits. Denote by $\theta(a,b)$ that part of the profit of producer b which is owned by consumer a. On the basis of the above, the share mapping $\theta : A\times B \to [0,1]$ satisfying

$$\forall b \in B \ \sum_{a\in A} \theta(a,b) = 1,$$

is defined.
Assume that the production plan realized by producer b are denoted by $\tilde{y}^b$. In this situation wealth w^a of consumer a is of the form:

$$w^a = p \circ \omega^a + \sum_{b\in B} \theta(a,b) \cdot (p \circ \tilde{y}^b). \tag{1}$$

Let

$$\omega = \sum_{a\in A} \omega^a \in \mathbb{R}^\ell. \tag{2}$$

Now the following definition can be formulated:

Definition 3. *Let $p \in \mathbb{R}^\ell$. The structure*

$$E_q = \left(\mathbb{R}^\ell, P_q, C_q, \ \theta, \omega\right),$$

is called the private ownership economy. If, for every $b \in B$, $\eta^b(p) \neq \emptyset$, $\tilde{y}^b = y^{b*}$ for a vector $y^{b*} \in \eta^b(p)$ as well as, for every $a \in A$ and w^a given by (1), sets $\beta^a(p)$ and $\varphi^a(p)$ are not empty, then the system E_q is called the Debreu economy and is denoted by E_p.

Vector (2) is called the total endowment of economy E_q. More about systems and quasi-systems, as well as about private ownership economies and their structures of action, the reader can find for example in Mas-Colell et al. (1995), Moore (2007), Lipieta (2013, 2015a, 2015b), Lipieta, Malawski (2016).
Consider a private ownership economy E_q. If $x^a \in X^a$, for every $a \in A$, and $y^b \in Y^b$, for every $b \in B$, then the sequence

$$\left(x^{a_1}, \ldots, \ x^{a_m}, \ y^{b_1}, \ldots, y^{b_n}\right) \in \left(\mathbb{R}^\ell\right)^{m+n} \tag{3}$$

is called the allocation. If additionally

$$\sum_{a\in A} x^a - \sum_{b\in B} y^b = \omega, \tag{4}$$

then the sequence (3) is called the feasible allocation. The equation (4) is called the market clearing condition.
Let $p \in \mathbb{R}^{\ell}$ be a given price vector in economy E_q. Suppose that every producer b realizes a production plan $y^{b*} \in \eta^b(p)$ maximizing his profit, every consumer a realizes one of his consumptions plans $x^{a*} \in \varphi^a(p)$ by w^a given by (1). If additionally allocation

$$(x^{a_1*}, \dots,\ x^{a_m*},\ y^{b_1*}, \dots, y^{b_n*})$$

is feasible, then it is said that economy E_p is in equilibrium. Consequently, the sequence

$$(x^{a_1*}, \dots,\ x^{a_m*},\ y^{b_1*}, \dots, y^{b_n*}, p) \in (\mathbb{R}^{\ell})^{m+n+1}$$

is called the state of equilibrium in economy E_p. In this situation vector p is called the equilibrium price vector and it is said that there is equilibrium in economy E_p or economy E_p is in equilibrium.

3 Adjustment processes in the private ownership economy

The private ownership economy as well as the Debreu economy are static models. Hence, to determine the changes in economic agents' activities, the idea of adjustment processes can be used.
Let $t = 0$ mean a starting point for the considered economic process and $\tau \in \{2, 3, \dots\}$. The point of time $t = \tau$ is a collusive ending point of the analysed process. Points of time $t = 1, \dots, \tau$ are identified with time intervals $[t-1, t)$ at which activities on markets of all economic agents are constant. Due to the above assumption, if an agent introduces changes in his economic activity, then it will be carried in one of the point $t \in \{1, \dots, \tau\}$. Saying "at time t", we mean "at time interval $[t-1, t)$" for $1 \leq t \leq \tau$.
The definition of the adjustment process is borrowed from Hurwicz (1987, p. 1442), however, it is slightly modified to enable the modelling of processes adjusting the activities of economic agents to equilibrium. Let $\mathrm{K} \in \{1, 2, \dots\}$ be the number of economic agents in the economy under study. Set $K = \{k_1, k_2, \dots, k_{\mathrm{K}}\}$ stands for the set of economic agents, set Z denotes the set of possible resource allocations.
Let $k \in K$. A set or sequence of characteristics determining agent k on markets at time t is called the environment of agent k at time t. By the fact that agents' economic activities are constant on the considered time intervals, we assume that the environment of every agent k is also constant at time t. The environment of agent k at time t is denoted by $e^k(t)$, whereas symbol $E^k(t)$ stands for the set of all his feasible environments at this time ($e^k(t) \in E^k(t)$). The set

$$E(t) \overset{\text{def}}{=} E^{k_1}(t) \times E^{k_2}(t) \times \dots \times E^{k_{\mathrm{K}}}(t)$$

is called the set of environments at time t, while the set

$$E = E(1) \times \cdots \times E(\tau)$$

is the set of environments.

The set of messages to be used on markets by agent k at time t is denoted by $M^k(t)$, the message of agent k at time t is denoted by $m^k(t)$, $m^k(t) \in M^k(t)$. It is assumed that messages $m^k(t)$ of every agent $k \in K$ are not changed at time t. Actually, message $m^k(t)$ consists of all signals and information which agent k shares with other agents within his activity on markets at time t (see Hurwicz, Reiter, 2006, p. 26-27). In competitive models, messages are often identified with agents' plan of action at given prices (see Hurwicz, Reiter, 2006, p. 36, 39). The set

$$M(t) \stackrel{\text{def}}{=} M^{k_1}(t) \times M^{k_2}(t) \times \cdots \times M^{k_{\mathrm{K}}}(t)$$

is called the set of messages at time t, vector

$$m(t) = \left(m^{k_1}(t),\ m^{k_2}(t),\ \ldots, m^{k_{\mathrm{K}}}(t)\right) \in M(t)$$

is called the message at time t, $m(t) \in M(t)$, the set

$$M = M(1) \times \cdots \times M(\tau)$$

is the set of messages. The sequence

$$m = (m\,(1)\,,\ \ldots,\ m(\tau))\,,$$

where $m(t) \in M(t)$, for $t = 1, \ldots, \tau$ is called the process of exchanging messages. It is said that process of exchanging messages is in equilibrium, if for some $t \in \{1, \ldots, \tau - 1\}$,

$$m\,(t) = m\,(t+1) = \cdots = \ m(\tau).$$

Message $m\,(t)$ satisfying the above condition is called the stationary message (compare to Hurwicz, 1987, p. 1442). The function

$$f_t^k : M(t) \times E(t) \ni (m\,(t)\,,\ e\,(t)) \to m^k\,(t+1) \in M^k(t+1),$$

which, at time t, assigns to the pair: environment $e\,(t)$ and message $m\,(t)$, the message $m^k\,(t+1)$ of agent k at time $t+1$, is called the response function of agent k at time t. The message $m^k\,(t+1) = f_t^k\,(m(t), e(t))$ consists of all information, which agent k characterized by environment $e^k(t)$ - coordinate k of environment $e(t)$, sends to other agents, characterized by adequate coordinates of vector $e\,(t)$, on the basis of the knowledge transformed by message $m(t)$. It is assumed that the value of the response function f_t^k at $(m(t), e(t))$ describes the activity of agent k on markets at time $t+1$. Now, we put the following definition:

Definition 4. *The structure*

$$(M, f, h), \tag{5}$$

where:

M *is the set of messages,*

$f = f_1 \times \cdots \times f_{\tau-1}$, *is called the response function in which*

$$f_t = \left(f_t^{k_1}, \ldots,\ f_t^{k_K}\right) : M(t) \times E(t) \to M(t+1)$$

is the response function at time t $(t = 1, \ldots,\ \tau - 1)$

$h = h_1 \times \cdots \times h_\tau$ *is the outcome function, in which*

$$h_t : M(t) \to Z(t)$$

is the outcome function at time $t = 1, \ldots, \tau$, *which to every message* $m(t) = \left(m^{k_1}(t), m^{k_2}(t), \ldots, m^{k_k}(t)\right)$ *assigns the allocations which are the result of analysis of the message* $m(t)$ *by economic agents,*

is called the adjustment process.
Number $\tau - 1$ *is called the number of steps of the adjustment process.*

Adjustment processes have been studied in the economic literature many times so far (compare to Marschak, 1972; Marschak, 1987, p.1389; Hurwicz, 1987, 1994; Lipieta, 2016). The definition of the adjustment process was inspired by the *tatonnement* process (see Hurwicz, 1987, p. 1441), in such a way that final outcomes of an adjustment process were determined, if an adequate process of exchanging messages was in equilibrium. It is easy to see that the *tatonnement* process is an adjustment process in the sense of Definition 4.
Let, for $t \in \{1, 2, \ldots,\ \tau\}$ and $b \in B$,

$Y^b\,(t)$ mean the set of plans of action of producer b, feasible to realization at time t,

$y^b(t)$ denote the plan of producer b realized at time t, $y^b(t) \in Y^b(t)$.

In the same way, the characteristics of consumers: $X^a\,(t)$ and $\epsilon_t\,(a)$ at time t, for $a \in A$, are defined. The correspondence of preference relations at time t is denoted by $\varepsilon_t\,(a) = \preccurlyeq_t^a$, where $\preccurlyeq_t^a\ \subset X^a\,(t) \times X^a\,(t)$ means the preference relation of consumer a at time t.
On the basis of the above notation, the environment $e^k\,(t)$ of every economic agent $k \in K = A \cup B$ at time t is defined. Namely

$$e^k(t) = \left(\tilde{y}_t(k), \widetilde{\chi}_t(k), \widetilde{\epsilon}_t(k), \widetilde{\varepsilon}_t(k), \widetilde{\theta}_t(k,\cdot)\right), \tag{6}$$

where:

$\tilde{y}_t(k) = Y^k(t)$ for $k \in B$, $\quad \tilde{y}_t(k) = \{0\}$ for $k \notin B$,

$\widetilde{\chi}_t(k) = X^k(t)$ for $k \in A$, $\quad \widetilde{\chi}_t(k) = \{0\}$ for $k \notin A$,

$\widetilde{\epsilon}_t(k) = \ \omega^k$ for $k \in A$, $\quad \widetilde{\epsilon}(k) = 0$ for $k \notin A$,

$\widetilde{\varepsilon}_t(k) = \ \preccurlyeq_t^a$ for $k \in A$, $\quad \widetilde{\varepsilon}(k) = \{\emptyset\}$ for $k \notin A$,

the mapping $\widetilde{\theta}_t : K \times K \to [0,1]$ satisfies for $a \in A$ and $b \in B$

$\widetilde{\theta}_t(k,\cdot) \equiv 0$ for $k \notin A$, $\widetilde{\theta}_t(\cdot,k) \equiv 0$ for $k \notin B$,

$\forall b \in B\ \sum_{a \in A} \widetilde{\theta}_t(a,b) = 1$,

the number $\widetilde{\theta}_t(a,b)$ is the share at time t of consumer for $a \in A$ in the profit of producer $b \in B$.

Let us notice that, for every $t = 1, \ldots, \tau$, components of the environment at time t

$$e(t) = \left(e^{k_1}(t),\ e^{k_2}(t),\ \ldots, e^{k_{\mathrm{K}}}(t)\right) \in E(t)$$

form in fact a private ownership economy. This economy will be denoted by $\mathrm{E}_q(t)$. Set of environments $E^k(t)$ of every agent $k \in K$ at time $t = 1, \ldots, \tau$ is of the form:

$$E^k(t) = P(\mathbb{R}^\ell) \times P(\mathbb{R}^\ell) \times \mathbb{R}^\ell \times P(\mathbb{R}^\ell \times \mathbb{R}^\ell) \times \mathcal{F}(K,[0,1]),$$

where

$$P(\Omega) = \{X : X \subset \Omega\},$$

for a set Ω, and

$$\mathcal{F}(K,[0,1]) \stackrel{\text{def}}{=} \left\{ \begin{array}{c} f|\ f : K \to [0,1],\quad f(k,\cdot) \equiv 0 \text{ for } k \notin A,\ f(\cdot,k) \equiv 0 \text{ for } k \notin B, \\ \forall b \in B\ \sum_{a \in A} f(a,b) = 1 \end{array} \right\}.$$

Assume that if $k \in B$, then set $\eta_t^k(p)$ means the set of optimal plans of producer k at time t; if $k \notin B$, then set $\eta_t^k(p) = \{0\}$. Similarly, if $k \in A$, then set $\varphi_t^k(p)$ means the set of optimal plans of consumer k at time t; if $k \notin A$, then $\varphi_t^k(p) = \{0\}$. Under the above arrangements, the message of every agent $k \in K$ at time $t = 1, \ldots, \tau$, can be of the form:

$$m^k(t) \stackrel{\text{def}}{=} \left(p(t), y^k(t), x^k(t)\right), \tag{7}$$

where:

$x^k(t) = 0 \in \mathbb{R}^\ell$ for $k \notin A$,

$x^k(t) \in X^k(t)$ for $k \in A$,

$y^k(t) = 0 \in \mathbb{R}^\ell$ for $k \notin B$,

$y^k(t) \in \eta_t^k(p)$ for $k \in B$.

Consequently, $M^k(t) = \mathbb{R}^\ell \times \mathbb{R}^\ell \times \mathbb{R}^\ell$.

We can see that message $m^k(t)$ of agent k at time t consists of his plans of action, realized at time t at given prices. Producers realize their optimal plans of actions and observe the demand and the products offered by other producers. Under the assumption of perfect competition producers are aware of the surpluses and deficiencies of the supply. Hence some of them may take a decision about modifying the amounts of inputs and outputs determining their activity. So, for every agent $k \in K$ and time $t \in \{1, \ldots, \tau - 1\}$, response function f_t^k assigns to the message $m(t) \in M(t)$ and environment $e(t) \in E(t)$, plans of action of agent k, realized at time $t+1$ at given prices, namely message $m^k(t+1)$ of the form:

$$m^k(t+1) = f_t^k(m(t), e(t)) \stackrel{\text{def}}{=} \left(p(t+1), y^k(t+1), x^k(t+1)\right). \tag{8}$$

It can be said that every agent k chooses his plans of action at time $t+1$ as a reply to price vector $p(t+1)$. Let

$$x(t) = (x^{a_1}(t), x^{a_2}(t), \ldots, x^{a_m}(t))$$

and

$$y(t) = \left(y^{b_1}(t), y^{b_2}(t), \ldots, y^{b_n}(t)\right).$$

On the basis of the above notation we define the set of outcomes at time $t = 1, \ldots, \tau - 1$:

$$Z(t) \stackrel{\text{def}}{=} \left\{ \begin{array}{c} \left((x^{a_1}(t), \ \ldots, x^{a_m}(t)), \ \left(y^{b_1}(t), \ldots, y^{b_n}(t)\right)\right) : \forall a \in A \\ x^a(t) \in X^a(t) \ \ \forall b \in B \ y^b(t) \in \eta_t^k(p) \ \ \sum_{a \in A} x^a(t) - \sum_{b \in B} y^b(t) = \omega(t) \end{array} \right\}$$

and

$$Z(\tau) \stackrel{\text{def}}{=} \left\{ \begin{array}{c} \left((x^{a_1}(\tau), \ \ldots, x^{a_m}(\tau)), \ \left(y^{b_1}(\tau), \ldots, y^{b_n}(\tau)\right)\right) : \forall a \in A \\ x^a(\tau) \in \varphi_\tau^k(p) \ \ \forall b \in B \ y^b(\tau) \in \eta_\tau^k(p) \ \ \sum_{a \in A} x^a(\tau) - \sum_{b \in B} y^b(\tau) = \omega(\tau) \end{array} \right\}.$$

The outcome function $h_t : M(t) \to Z(t)$ at time $t = 1, \ldots, \tau$:

$$\begin{aligned} h_t(m(t)) &= h_t\left(m^{k_1}(t), \ldots, m^{k_k}(t)\right) = \\ &= h_t\left(\left(p(t), y^{k_1}(t), x^{k_1}(t)\right), \ldots, \left(p(t), y^{k_k}(t), x^{k_k}(t)\right)\right) \\ &\stackrel{\text{def}}{=} \left((x^{a_1}(t), x^{a_2}(t), \ldots, x^{a_m}(t)), \left(y^{b_1}(t), \ y^{b_2}(t), \ldots, y^{b_n}(t)\right)\right) = \\ &= (x(t), y(t)). \end{aligned} \tag{9}$$

Loosely speaking, function h_t assigns message $m(t)$, the sequence of action of economic agents transferred by this messages. If message $m(t)$, for a $t \in \{1, \ldots, \tau - 1\}$, is the stationary message, then $p(t) = p(t+1)$, and state

$$(x(t), y(t), p(t))$$

is the state of equilibrium in economy E_p (compare to Hurwicz, 1987).

Definition 5. *It is said that adjustment process* (5)

1. *requires the involvement of agent* $k \in K$ *if, for some* $t \in \{1, \ldots, \tau - 1\}$, $e^k(t) \neq e^k(t+1)$ *or* $m^k(t) \neq m^k(t+1)$,

2. *is the adjustment process in a private ownership economy, if components of an environment at time* $t = 1$

$$e(1) = \left(e^{k_1}(1),\ e^{k_2}(1),\ \ldots, e^{k_k}(1)\right) \in E(1)$$

form a private ownership economy,

3. *is the adjustment process in a Debreu economy, if components of an environment at time* $t = 1$

$$e(1) = \left(e^{k_1}(1),\ e^{k_2}(1),\ \ldots, e^{k_k}(1)\right) \in E(1)$$

form a Debreu economy,

4. *results in equilibrium in a Debreu economy, if components of an environment at time* $t = \tau$

$$e(\tau) = \left(e^{k_1}(\tau),\ e^{k_2}(\tau),\ \ldots, e^{k_k}(\tau)\right) \in E(\tau)$$

form a Debreu economy in equilibrium.

Let an adjustment process in a Debreu economy E_p be given ($\mathrm{E}_p = \mathrm{E}_p(1)$) and $\tau \in \{2, 3, \ldots\}$. Consider an adjustment process in economy $\mathrm{E}_p(1)$. Assume that at every time $t \in \{1, \ldots, \tau\}$, the components of environment $e(t)$ form the private ownership economy $\mathrm{E}_q(t)$. Every economy $\mathrm{E}_q(t)$ is called the transformation of the initial economy $\mathrm{E}_p(1)$, which will be shortly noted by $\mathrm{E}_p(1) \subset \mathrm{E}_q(t)$. The economy $\mathrm{E}_q(\tau)$ is called the final transformation of economy $\mathrm{E}_q(1)$. It may appear that, at some time $t \in \{1, \ldots, \tau\}$, the private ownership economy $\mathrm{E}_q(t)$ is the Debreu economy, which will be denoted by $\mathrm{E}_q(t) = \mathrm{E}_p(t)$. Moreover, in every transformation of the initial economy, the set of commodities as well as the set of economic agents are the same.

The adjustment process in a Debreu economy can be used to model a procedure of adjusting of producers' or consumers' plans of action as well as prices of goods to equilibrium, without changing the set of commodities and the set of economic agents. That process can be also viewed as the adapting process in the Andersen's meaning (see Andersen, 2009), during which the economic agents adapt new technologies, which results in a new state of equilibrium in the final transformation of the economy under study.

4 Adjustment processes in Debreu economy resulting in equilibrium

We start this part of the paper from the following, simple example:

Example 1. *Consider a private ownership economy with two commodities, two producers and two consumers. The characteristics of economic agents are the following:*

$$Y^1 = \{(y_1,\ y_2) \in \mathbb{R}^2 :\ y_2 \le -2y_1 + 4 \wedge\ y_2 \le 2\},$$

$$Y^2 = (-\infty, 1] \times (-\infty, -1], X^1 = [0, \infty) \times [0, \infty)\,,$$

$$X^2 = [0, 4] \times [0, \infty)$$

$$\omega^1 = (1, 3)\,, \omega^2 = (2, 2)\,,$$

$$(x_1,\ x_2) \preccurlyeq^1 (\tilde{x}_1,\ \tilde{x}_2) \Longleftrightarrow \max\{x_1,\ x_2\}\ \le \max\{\tilde{x}_1,\ \tilde{x}_2\}\ ,$$

$$(x_1,\ x_2) \preccurlyeq^2 (\tilde{x}_1,\ \tilde{x}_2) \Longleftrightarrow x_1 - x_2 \le \tilde{x}_1 - \tilde{x}_2, ,$$

$$\theta\,(2, 1) = \frac{1}{2}, \theta\,(1, 2) = 1.$$

We show that there is no equilibrium in that economy.

Solution. It is not difficult to check that if an equilibrium price vector $p = (p_1, p_2)$ exists, then their coordinates should satisfy two conditions $p_1 \ge 0$ and $p_2 \ge \frac{1}{2}\ p_1$. Moreover, if $p = (p_1, p_2)$ is the equilibrium price vector, then for every $k > 0$, vector $kp = (kp_1, kp_2)$ is also the equilibrium price vector.
Firstly, we show that there is no equilibrium in the considered economy at price vector $p = (2, 1)$. At given prices, we get: $y^{1*} = (y_1,\ -2y_1 + 4)$ for $y_1 \ge 1$, $y^{2*} = (1, -1)$, $w^1 = w^2 = 4p_1$ (see (1)), $x^{1*} = (0, 8)$, $x^{2*} = (4, 0)$, (see (2)). However, for the above optimal plans, the market clearing condition (4) is not satisfied, which means that neither $p = (2, 1)$ nor vector $\left(p_1, \frac{1}{2}\ p_1\right)$, for every $p_1 > 0$, is the equilibrium price vector in the above economy.
Consider vectors $p = (0, p_2)$ for $p_2 \ge 0$. In this case $\varphi^1(p) = \emptyset$ and there is no equilibrium in the economy under study.
At the end, we show that $p = (p_1, p_2)$, where $p_1 > 0$ and $p_2 > \frac{1}{2}\ p_1$ is not the equilibrium price vector. In that case $y^{1*} = (1, 2)$, $y^{2*} = (1, -1)$, $x_2^{2*} = 0$, $x^{1*} = \left(\frac{5}{2} + 3 \cdot \frac{p_2}{p_1}, 0\right)$ if $p_2 > p_1$ or $x^{1*} = \left(0,\ \frac{5}{2} \cdot \frac{p_1}{p_2} + 3\right)$ if $p_2 < p_1$, as well as $x^{1*} \in \left\{\left(\frac{11}{2}, 0\right),\ \ (0,\ \frac{11}{2})\right\}$ if $p_2 = p_1$.
If condition (4) were satisfied, then $x^{1*} + x^{2*} = \omega + y^{1*} + y^{2*} = (5, 6)$ and consequently $x_2^{1*} = 6$. It would mean that $x^{1*} = \left(0,\ \frac{5}{2} \cdot \frac{p_1}{p_2} + 3\right)$ and $p_2 < p_1$. By the previous $\frac{5}{2} \cdot \frac{p_1}{p_2} + 3 = 6 \Rightarrow \frac{p_1}{p_2} = \frac{6}{5}$. Then, by (4), $x_1^{2*} = 5$, which contradicts the definition of the set X^2. □

Let us notice that in Example 1 some assumptions of the First Existence Theorem for Competitive Equilibrium (Arrow, Debreu, 1954, p. 266) are not satisfied i. e. set $Y = Y^1 + Y^2$ does not satisfy assumptions I.b, I.c (*ibid.* p. 267), set Y^2 does not satisfy assumption I.a (*ibid.* p. 267), the utility function determined by preference relation $\preccurlyeq^1$ does not satisfy assumption III. c, (*ibid.* p. 269). Example 1 elucidates that in some cases, equilibrium in a private ownership economy cannot be achieved. In the spirit of the activity analysis, namely the study of interactions between inputs and outputs of production (see for instance Koopmans, 1951), we will concentrate on some possible changes in the producers' activities necessary for the existence of equilibrium in a transformation of an initial private ownership economy. In this context, a procedure of a moderate change of the production sector of a private ownership economy is presented in which, under some natural economic assumptions, at given prices, equilibrium will be achieved.

Let a Debreu economy E_p be given. Suppose that at given price system $p \in \mathbb{R}^\ell$ every producer $b \in B$ could realize a production plan $y^{b*} \in \eta^b(p)$ maximizing his profit as well as for every $a \in A$ there exists a consumption plan $x^{a*} \in \varphi^a(p)$ for w^a given by (1). Suppose that allocation

$$(x^{a_1*}, \dots,\ x^{a_m*},\ y^{b_1*}, \dots, y^{b_n*}) \in \left(\mathbb{R}^\ell\right)^{m+n} \tag{10}$$

is not feasible. In consequence, the sequence

$$(x^{a_1*}, \dots,\ x^{a_m*},\ y^{b_1*}, \dots, y^{b_n*}, p) \in \left(\mathbb{R}^\ell\right)^{m+n+1}$$

is not the state of equilibrium in economy E_p. Let

$$x^* = \sum_{a \in A} x^{a*}, y^* = \sum_{b \in B} y^{b*}, \omega = \sum_{a \in A} \omega^a.$$

Hence

$$\zeta \overset{\mathrm{def}}{=} x^* - y^* - \omega \neq 0. \tag{11}$$

Assume that for ζ given by (11), Walras Law (see for instance Mas-Colell et al., 1995) is satisfied, namely

$$p \circ \zeta = 0.$$

We show that p is the equilibrium price vector in a transformation of economy E_p, in which the production system is changed through an adjustment process, in such a way, that consumers plans of action and consequently their optimal plans are not changed.

Theorem 1. *If*

$$\exists \tau \in \{2, 3, \dots\}\ \forall t \in \{1, \dots, \tau - 1\} :\ x^* - \frac{t}{\tau - 1} \cdot \zeta \in X^{a_1} + \dots + X^{a_m} \tag{12}$$

as well as

$$\exists B_0 \subset B: \textit{ vectors } \zeta \textit{ and } \sum_{b\in B_0} y^{b*} \textit{ are linearly independent,} \tag{13}$$

then there is equilibrium in a transformation $E_p(\tau)$ of economy E_p, in which the production sector is modified through an adjustment process of the form (5) consisting of $\tau - 1$ steps. Moreover, for $t \in \{1, \ldots, \tau - 1\}$, consumption system $C_q(t+1)$ in economy $E_q(t+1)$ is the same as in economy E_p ($C_q(t+1) = C_q$) as well as every producer from set B_0 changes his productive activities in the same way.

Proof. Consider $B_0 \subset B$ satisfying (13). Vectors ζ and $\sum_{k\in B_0} y^{k*}$ are linearly independent, so there exists a vector $h \in \mathbb{R}^{\ell}\backslash\{0\}$ satisfying

$$\begin{cases} h \circ \left(\zeta + \sum_{k\in B_0} y^{k*}\right) = 0 \\ h \circ \zeta = 1. \end{cases} \tag{14}$$

Put for $t \in \{1, \ldots, \tau - 1\}$:

$Y^k(t+1) = Y^k(t) \cup \left\{ y^k(1) - t/(\tau-1) \cdot (h \circ y^k(1)) \cdot \zeta : \ y^k(1) \in Y^k(1) \right\}$ for $k \in B_0$,

$Y^k(t+1) = Y^k(1)$ for $k \in K\backslash B_0$,

$X^k(t+1) = X^k(1)$, $\preccurlyeq^k_{t+1} = \preccurlyeq^k_1$, $\omega^k(t+1) = \omega^k(1)$, for every $k \in K$.

$\widetilde{\theta}_{t+1} = \widetilde{\theta}_1$.

Let, for every $k \in K$ and $t \in \{1, \ldots, \tau\}$, environment $e^k(t)$ be of the form (6), message $m^k(t)$ be of the form (7). Hence

$$m(t) = \left(\left(p(t),\ y^{k_1}(t),\ x^{k_1}(t)\right), \ldots,\ \left(p(t),\ y^{k_K}(t),\ x^{k_K}(t)\right)\right),$$

where $p(1) = p$. For every $k \in K$ and $t = 1, \ldots,\ \tau - 1$, we define mappings

$$f^k_t : M(t) \times E(t) \ni (m(t), e(t)) \to m^k(t+1) \in M^k(t+1),$$

$t \in \{1, \ldots, \tau - 1\}$, by the formula:

$$f^k_t\left(m(t), e(t)\right) = \left(p,\ y^k(t+1),\ x^k(t+1)\right) \text{ for } k \in B,$$

where:

$y^k(t+1) = y^k(1) - t/(\tau-1) \cdot \left(h \circ y^k(1)\right) \cdot \zeta$, for $k \in B_0$,

$y^k(t+1) = y^k(1)$, for $k \in B\backslash B_0$,

$x^k(t+1) \in X^k$, for every $t \in \{0, \ldots, \tau-2\}$, is the realized consumption plan of consumer $k \in A$ at time $t+1$, such that, for every $t \in \{1, \ldots, \tau-2\}$,

$$\sum_{k\in A} x^k\,(t+1) = x^* - \frac{t}{\tau-1}\cdot\zeta,$$

$x^k\,(\tau) = x^{k*}$, for $k \in A$.

Hence, for $t \in \{1, \ldots, \tau-1\}$,

$$y^k\,(t+1) = \begin{cases} y^k\,(1) - t/(\tau-1)\cdot\big(h\circ y^k\,(1)\big)\cdot\zeta & \text{for} \quad k \in B_0 \\ y^k\,(1) & \text{for} \quad k \notin B_0, \end{cases},$$

$x^k(t+1) = 0$ for $k \notin A$,

$y^k(t+1) = 0$ for $k \notin B$.

Since Walras Law is satisfied, then, for every $k \in B_0$,

$$p\circ y^k\,(t+1) = p\circ\left(y^k - \frac{t}{\tau-1}\cdot(h\circ y^k)\cdot\zeta\right) = p\circ y^k - \frac{t}{\tau-1}\cdot\big(h\circ y^k\big)\cdot(p\circ\zeta) = p\circ y^k.$$

By the above, where

$$y^k\,(t+1) \stackrel{\text{def}}{=} \begin{cases} y^{k*} - t/(\tau-1)\cdot(h\circ y^{k*})\cdot\zeta & \text{for} \quad k \in B_0 \\ y^{k*} & \text{for} \quad k \notin B_0 \end{cases}$$

is the optimal plan of producer k ($y^{k*}\,(t+1) = y^k\,(t+1) \in \eta^k_{t+1}\,(p)$).
Hence producers' maximal profits, the budget sets and consequently the optimal consumers' plans in economies $\mathrm{E}_p\,(1)\,,\ \ldots, \mathrm{E}_q(\tau)$ remain the same. Consequently, every economy $\mathrm{E}_q\,(t+1)$, for $t = 1, \ldots, \tau-1$, is the Debreu economy ($\mathrm{E}_q\,(t+1) = \mathrm{E}_p\,(t+1)$).
By (12), for $t = 1, \ldots, \tau-1$,

$$\sum_{k\in K} y^{k*}(t+1) + \omega = y^* + \frac{\cdot t}{\tau-1}\cdot\zeta + \omega = x^* - \left(1 - \frac{\cdot t}{\tau-1}\right)\cdot\zeta \in X^{a_1} + \cdots + X^{a_m},$$

which gives that allocation

$$\big(x^{a_1}(t+1), \ldots,\ x^{a_m}(t+1),\ y^{b_1}(t+1), \ldots, y^{b_n}(t+1)\big)\,, \tag{15}$$

is feasible in economy $\mathrm{E}_p\,(t+1)$. However, above all,

$$\sum_{k\in K} x^{k*}\,(t+1) - \sum_{k\in K} y^{k*}\,(t+1) - \omega = \left(1 - \frac{\cdot t}{\tau-1}\right)\cdot\zeta \,\text{for}\, t \in \{1, 2, \ldots, \tau-1\},$$

which means that sequence

$$\left(x^{a_1*},\ldots,\ x^{a_m*},\ \tilde{y}^{b_1*},\ldots,\tilde{y}^{b_n*}\right)\in\left(\mathbb{R}^{\ell}\right)^{m+n}, \tag{16}$$

where

$$\tilde{y}^{k*}=\begin{cases} y^{k*}-(h\circ y^{k*})\cdot\zeta & \text{for} \quad k\in B_0 \\ y^{k*} & \text{for} \quad k\in B\backslash B_0 \end{cases}$$

is the state of equilibrium in economy $\mathrm{E}_p(\tau)$. The above ends the proof. □

As we can see by Theorem 1, equilibrium in economy $\mathrm{E}_p(\tau)$ – the transformation of economy E_p can be achieved after $\tau-1$ steps if, for a given sequence of optimal plans (10):

1. Walras Law is satisfied at given prices,
2. there exist some producers for whose vector ζ and the sum of optimal plans of producers from the given set are not linearly dependent (assumption (13)),
3. total sum of given optimal consumers' plans can be realized, for every $t\in\{1,\ldots,\tau-1\}$, in the set $X^{a_1}+\cdots+X^{a_m}+\{t/(\tau-1)\cdot\zeta\}$ (assumption (12)).

During the adjustment process defined in the proof of Theorem 1, the producers from set B_0 have to change their productive activities in the same way.
The adjustment process defined in the proof of Theorem 1 can be organized as follows. Firstly, a manager of production sphere determines some producers satisfying assumption (13). The chosen producers, for instance, the producers using harmful technologies, will have to change their productive activity. Secondly, the manager decides what properties production plans of the selected producers should have (see also Lipieta, 2015a, pp 192,193) at the ending point of the given process, through the choice of one of vectors satisfying the condition (14). For instance, he could enforce, by introducing adequate rules, the use of only pro-ecological technologies at that period. Thirdly, the manager determines a starting point, an ending point as well as intermediate points, namely points of time in which producers will introduce changes in their activities. Within the adjustment process, the distance in the given metric between realized allocation (15) and the state of equilibrium (16) decreases with time. According to Theorem 1 at the ending point, allocation (15) takes the form (16).
Let us present how the adjustment process defined in the proof of Theorem 1 can be employed to the economy defined in Example 1.

Example 2. *1) We show that the assumptions of Theorem 1 are satisfied in the economy defined in Example 1 with price vector $p=(2,1)$.*
We recall that, at price vector $p=(2,1)$: $x^{1*}=(0,8)$, $x^{2*}=(4,0)$,

$y^{1*} = (y_1,\ -2y_1 + 4)$ for $y_1 \geq 1$, $y^{2*} = (1, -1)$ but allocation $(x^{1*}, x^{2*}, y^{1*}, y^{2*})$ is not feasible. Moreover,

(a) $\zeta = x^{1*} + x^{2*} - \left(y^{1*} + y^{2*}\right) - \left(\omega^1 + \omega^2\right) = (4, 8) - (y_1 + 1,\ -2y_1 + 3) + - (3, 5) = (-y_1, 2y_1)$ *for* $y_1 \geq 1$*;*

(b) $\zeta \neq 0$ *for every* $y_1 \geq 1$

(c) the Walras Law is satisfied for every $y_1 \geq 1$*,*

(d) $\forall y_1 \in [1, 4]\ \forall \tau \in \{2, 3, \dots\}\ \ \forall t \in \{1, \dots, \tau - 1\}\ \ x^* - \frac{t}{\tau - 1} \cdot \zeta \in X^1 + X^2 = = X^1$*,*

(e) $y^{2*} = (1, -1)$ *and* $\zeta = (-y_1, 2y_1)$*, for every* $y_1 \geq 1$*, are linearly independent, hence* $B_0 = \{b_2\}$.

Conditions (1) – (5) *give that, for* $y_1 \in [1, 4]$*, the assumptions of Theorem 1 are satisfied.*

2) We model one of the adjustment processes defied in the proof of Theorem 1. Put $y_1 = 1$*. Then* $y^{1*} = (1, 2)$*,* $\zeta = (-1, 2)$ *. and consequently*

$$\zeta + \sum_{b \in B_0} y^{b*} = (-1, 2) + (1, -1) = (0, 1).$$

For every $\tau \in \{2, 3, \dots\}$ *and for every* $t \in \{1, \dots, \tau - 1\}$

$$x^* - \frac{t}{\tau - 1} \cdot \zeta = (4, 8) - \frac{t}{\tau - 1} \cdot (-1, 2) \in X^1 = [0, \infty) \times [0, \infty).$$

It is easy to check that vector $h = (-1, 0)$ *satisfies system of equalities* (14)*. Put* $\tau = 2$*.*

On the basis of the proof of Theorem 1, we get that in the transformation of the economy defined in Example 1 in which only set $Y^2\,(1)$ *is transformed to set*

$$Y^2\,(2) \stackrel{\text{def}}{=} Y^2\,(1) \cup \left\{y^2 - \left((-1, 0) \circ y^2\right) \cdot (-1, 2) :\ y^2 \in Y^2\,(1)\right\} =$$
$$= \{(y_1,\ y_2) \in \mathbb{R}^2 :\ y_2 \leq -2y_1 + 1 \wedge\ y_2 \leq 1\},$$

there is the state of equilibrium at price vector $p = (2, 1)$ *in which* $x^{1*} = (0, 8)$*,* $x^{2*} = (4, 0)$*,* $y^{1*} = (1, 2)$*,* $y^{2*} = (0, 1)$*.*

Now, let us present more remarks on Theorem 1. If $Y^{b_0}\,(t + 1) \not\subset Y^{b_0}\,(t)$, for $t \in \{1, \dots, \tau - 1\}$ and $b_0 \in B_0$, then producer $b_0 \in B_0$ operating within the framework of the given adjustment process has to introduce new technologies into his production activity such that every plan from the set $Y^{b_0}\,(t + 1) \setminus Y^{b_0}\,(t)$ would be feasible at time t (see Lipieta, 2013). The production set $Y^{b_0}(t + 1)$ in the economy $\mathrm{E}_q(t + 1)$ can be also reduced to the set

$$\tilde{Y}^{b_0}(t + 1) = \left\{y^{b_0 *} - \frac{t}{\tau - 1} \cdot \left(h \circ y^{b_0 *}\right) \cdot \zeta\right\}$$

for $t \in \{1, \dots, \tau - 1\}$. The set $\tilde{Y}^{b_0}(\tau)$ is the linear production set contained in the space $\left\{x \in \mathbb{R}^\ell : h \circ x = 0\right\}$. If $\tilde{Y}^{b_0}(\tau) \subset Y^{b_0}(1)$, then the adjustment process defined in the proof of Theorem 1 relies only on the replacement the plan $y^{b_0 *}$ with the plan

$$y^{b_0 *}(\tau) = y^{b_0 *} - (h \circ y^{b_0 *}) \cdot \zeta.$$

Set B_0 satisfying assumption (13), also should be chosen before the beginning of the process. It can be done by a manager. In some cases there are several, or even infinitely many, numbers satisfying condition (12), hence the number of steps of the introduced adjustment process should be chosen by a manager established by producers from set B_0 before the beginning of an adjustment process. If τ increases, then the distance between vectors y^b and $y^b - 1/(\tau - 1) \cdot (h \circ y^b) \cdot \zeta$, in a given metric, will decrease, which in consequence means that the introduced changes are smaller and smaller. By the above, we also see that the adjustment process defined in Theorem 1 requiring involvement of producers from set B_0.
Let us also notice that under the assumptions of Theorem 1 and given τ satisfying (12), if $\ell > 2$, there exist infinitely many adjustment processes throughout which the initial economy $\mathrm{E}_p(1)$ is transformed to economy $\mathrm{E}_p(\tau)$ in which equilibrium exists. This is due to the existence of infinitely many vectors $h \in \mathbb{R}^\ell$ satisfying (14).
The lack of equilibrium in the initial economy is the sufficient condition to make producers change their activities on the market. Moreover, the set B_0 does not have to be unique, which increases the set of adjustment processes giving an equilibrium in economy $\mathrm{E}_p(\tau)$. Leadership or coordination of producers activities may be necessary to enforce the application one of the adjustment processes defined in the proof of Theorem 1. In the opposite case, equilibrium in the final transformation is unlikely to appear, unless the producers follow the same adjustment process.
Now we present a construction of an adjustment process resulting in equilibrium in a Debreu economy which can be applied if condition (13) is not satisfied.

Theorem 2. *If condition* (12) *is satisfied as well as* (13) *is not satisfied, then there is equilibrium in a transformation $E_p(\tau)$ of economy E_p, in which the production sector is modified through an adjustment process of the form* (5) *consisting of $\tau - 1$ steps. Moreover, for every $t \in \{1, \dots, \tau - 1\}$, consumption system $C_q(t+1)$ in economy $E_q(t+1)$ is the same as in economy E_p $(C_q(t+1) = C_q)$ as well as every producer from set B_0 can change his productive activities in the same way.*

Proof. If condition (13) is not satisfied then

$$\forall B_0 \subset B,\ B_0 \neq \emptyset: \text{ vectors } \zeta \text{ and } \sum_{b \in B_0} y^{b*} \text{ are linearly dependent.}$$

By the above, for every $k \in B$, vectors $y^{k*}$and ζ are linearly dependent. Hence there exists $c_k \in \mathbb{R} \backslash \{0\}$ such that

$$y^{k*} = c_k \cdot \zeta \ \text{ or } \ y^{k*} = 0 \tag{17}$$

By the above, $p \circ y^{k*} = 0$ for every $k \in B$. Assume firstly that

$$\exists B_0 \subset B : \sum_{k \in B_0} y^{k*} \neq 0. \tag{18}$$

Then

$$\exists s \in \mathbb{R} \backslash \{0\} : \ \zeta = s \cdot \sum_{k \in B_0} y^{k*}. \tag{19}$$

Consider a subset $B_0 \subset B$ satisfying (18). In such a case, the producers' response functions at time t are given by:

$$f_t^k\left(m(t), e(t)\right) = \left(p,\ y^k\left(1\right),\ x^k\left(t+1\right)\right) \text{ for } k \in B \backslash B_0,$$

$$f_t^k\left(m(t), e(t)\right) = \left(p,\ y^k\left(1\right) + \tfrac{t \cdot s}{\tau - 1} \cdot \overline{y}^k(1),\ x^k(t+1)\right) \text{ for } k \in B_0,$$

$t \in \{1, \ldots, \tau - 1\}$, while the production sets are of the form:

$$Y^k(t+1) = Y^k(1) \cup \left\{ y^k(1) + \tfrac{t \cdot s}{\tau - 1} \cdot \overline{y}^k(1) :\ y^k(1) \in Y^k(1) \right\} \text{ for } k \in B \backslash B_0$$

$$Y^k(t+1) = Y^k(1) \text{ for } k \in B_0,$$

for $t \in \{1, \ldots, \tau - 1\}$. Additionally,

$x^k(t+1) \in X^k$, for every $t \in \{0, \ldots, \tau - 2\}$ and $k \in A$, is the realized consumption plan of consumer $k \in A$ at time $t+1$, moreover,

$$\sum_{k \in A} x^k\left(t+1\right) = x^* - \frac{t}{\tau - 1} \cdot \zeta,$$

$x^k\left(\tau\right) = x^{k*}$, for $k \in A$,

$x^k(t+1) = 0$ for $k \notin A$.

The sequence

$$\left(x^{a_1 *}, \ldots,\ x^{a_m *},\ \tilde{y}^{b_1 *}, \ldots, \tilde{y}^{b_n *}\right) \in \left(\mathbb{R}^\ell\right)^{m+n},$$

where

$$\tilde{y}^{k*} = \begin{cases} (1+s) \cdot y^{k*} & \text{for} \quad k \in B_0 \\ y^{k*} & \text{for} \quad k \in B \backslash B_0 \end{cases}$$

is the state of equilibrium in economy $\mathrm{E}_p\left(\tau\right)$.
If condition (18) is not satisfied then, for every $k \in B$, $y^{k*} = 0$. For a $B_0 \subset B$, numbers $s_k > 0$, for $k \in B_0$, satisfying $\sum_{k \in B_0} s_k = 1$ can be chosen. Then

$$f_t^k\left(m(t), e(t)\right) = \left(p,\ y^k\left(1\right),\ x^k\left(t+1\right)\right) \text{ for } k \in B \backslash B_0,$$

$$f_t^k\left(m(t),e(t)\right) = \left(p, y^k(1) + t\cdot s_k/(\tau-1)\cdot\zeta,\ x^k(t+1)\right) \text{ for } k\in B_0,$$

while the production sets are of the form:

$$Y^k\left(t+1\right) = Y^k\left(1\right) + \{t\cdot s_k/(\tau-1)\cdot\zeta\} \text{ for } k\in B\backslash B_0,$$

$$Y^k\left(t+1\right) = Y^k(1) \text{ for } k\in B_0.$$

The sequence

$$\left(x^{a_1*},\ldots,\ x^{a_m*},\ \tilde{y}^{b_1*},\ldots,\tilde{y}^{b_n*}\right)\in\left(\mathbb{R}^\ell\right)^{m+n},$$

where

$$\tilde{y}^{k*} = \begin{cases} y^{k*} + s_k\cdot\zeta & \text{for} \quad k\in B_0 \\ y^{k*} & \text{for} \quad k\in B\backslash B_0 \end{cases}$$

is the state of equilibrium in economy $\mathrm{E}_p\left(\tau\right)$.
If, for every $k\in B_0$, $s_k = \frac{1}{n_0}$, where n_0 denotes the number of producers from set B_0, then producers from set B_0 will change their production activities in the same way. The rest of the proof goes in the same way as the proof of Theorem 1. □
As we can see by Theorem 2, equilibrium in $\mathrm{E}_p(\tau)$ – the transformation of economy E_p can be also achieved after $\tau-1$ steps if, for the given sequence of optimal plans (10):

1. Walras Law is satisfied at given prices,
2. for every subset of producers, sum of their optimal plans and vector ζ are linearly dependent (assumption (13) is not satisfied),
3. total sum of given optimal consumers' plans is feasible, for every $t\in\{1,\ldots,\tau-1\}$, in the set $X^{a_1}+\cdots+X^{a_m}+\{t/(\tau-1)\cdot\zeta\}$ (assumption (12)).

If $n>1$, then many possibilities of the choice of set B_0 as well as many numbers satisfying condition (12) occur. Hence, then there are many adjustment processes leading to equilibrium in a private ownership economy with modified production system according to the recipe defined in the proof of Theorem 2, in which every producer from set B_0 change his productive activities in the same way.
Adjustment process defined in the proof of Theorem 2 requires involvement of producers from set B_0. As earlier, every transformation of economy E_p is the Debreu economy.
The choice of set B_0, the number of steps as well as the arrangement of the rules of the adjustment process should be done by a manager before the beginning of the process. In difference to the adjustment process defined in the proof of Theorem 1, if in sequence (10), $y^{b_0*}\neq 0$ for some b_0, then equilibrium in the final transformation could be achieved due to a proper proportional increase of the amounts of inputs

and outputs in process y^{b_0*}. Similarly, more producers could modify their activities provided that they satisfy condition (18). In that case each of them also has to realize a plan proportional to ζ (see (11)). If, for every $b \in B$, in sequence (10), $y^{b*} = 0$, then according to the proof of Theorem 2, the manager could select any producers to change market activities. Equilibrium will appear if the chosen producers, in each single step of the adjustment process, realize a proper plan proportional to ζ. The adjustment process defined in the proof of Theorem 2 could be applied if equilibrium is not established on markets of "similar" commodities. In such case, disequilibrium might be caused by producers, who produce too much or too little similar or the same goods or use too much or too little similar or the same commodities in production processes. The previous statement means that condition (19) is satisfied.
Let us emphasize that Theorems 1 and 2 gives a recipe for adjustment the production sector of a Debreu economy to equilibrium under the assumption that Walras Law is satisfied at given prices (see (12)). In the analysed adjustment processes, every producer from set B_0 can change his productive activities in the same way.
Below a next adjustment process, in which producers can differently modify their activities on the market, is presented.

Theorem 3. *If condition* (12) *is valid, then there is equilibrium in a transformation* $E_p(\tau)$ *of economy* E_p, *in which the production sector is modified through an adjustment process consisting of* $\tau - 1$ *steps as well as, for* $t \in \{1, \dots, \tau - 1\}$, *consumption system* $C_q(t+1)$ *in economy* $E_q(t+1)$ *is the same as in economy* E_p *as well as some producers can modify their production activities in different ways.*

Proof. It is similar to the proof of Theorem 2. Let $B_0 \subset B$, $B_0 \neq \emptyset$. Assume that numbers $s_k > 0$, for $k \in B_0$, satisfying $\sum_{k \in B_0} s_k = 1$ are given. Every number s_k determines changes in market activity of producer k at time t. The proof of the theorem differs from the proof of Theorem 1 in the definitions of some response functions and some production sets at time t. The response functions at time t of producers are given by:

$$f_t^k(m(t), e(t)) = (p,\ y^k(t),\ x^k(t+1)) \text{ for } k \in B \backslash B_0,$$

$$f_t^k(m(t), e(t)) = (p,\ y^k(t) + s_k/(\tau - 1) \cdot \zeta,\ x^k(t+1)) \text{ for } k \in B_0,$$

$t \in \{1, \dots, \tau - 1\}$, while the production sets are of the form:

$$Y^k(t+1) = Y^k(t) \cup \left\{ y^k(t) + s_k/(\tau - 1) \cdot \zeta :\ y^k(t) \in Y^k(t) \right\} \text{ for } k \in B \backslash B_0,$$

$$Y^k(t+1) = Y^k(1) \text{ for } k \in B_0,$$

for $t \in \{1, \dots, \tau - 1\}$. As earlier

$x^k(t+1) \in X^k$, for every $t \in \{0, \dots, \tau-2\}$ and $k \in A$, is the realized consumption plan of consumer $k \in A$ at time $t+1$; moreover,

$$\sum_{k \in A} x^k(t+1) = x^* - \frac{t}{\tau - 1} \cdot \zeta,$$

$x^k(\tau) = x^{k*}$, for $k \in A$.

The sequence

$$(x^{a_1 *}, \dots, x^{a_m *}, \tilde{y}^{b_1 *}, \dots, \tilde{y}^{b_n *}) \in \left(\mathbb{R}^\ell\right)^{m+n},$$

where

$$\tilde{y}^{k*} = \begin{cases} y^{k*} + s_k \cdot \zeta & \text{for} \quad k \in B_0 \\ y^{k*} & \text{for} \quad k \in B \backslash B_0 \end{cases}$$

is the state of equilibrium in economy $\mathrm{E}_p(\tau)$. □

By Theorem 3, equilibrium in $\mathrm{E}_p(\tau)$ – the transformation of economy E_p can be achieved after $\tau - 1$ steps if, for the given sequence of optimal plans (10):

1. Walars Law is satisfied at given prices,
2. total sum of given optimal consumers' plans is feasible, for every $t \in \{1, \dots, \tau - 1\}$, in the set $X^{a_1} + \dots + X^{a_m} + \{t/(\tau-1) \cdot \zeta\}$ (assumption (12)).

During the adjustment process defined in the proof of Theorem 3, the producers from set B_0 do not have to change their productive activities in the same way. Let us emphasize that the adjustment process defined in the proof of Theorem 3 leads to equilibrium even if condition (13) is satisfied, but it requires other modification of the production sector of economy E_p than that defined in the proof of Theorem 1. A manager of the adjustment process defined in the proof of Theorem 3 should determine a number of steps, an initial point, an ending point and intermediate points as well as select producers who will change their production. The changes introduced by chosen producers, in contrast to the earlier analysed adjustment processes, are based on realization within the same period of two production plans: the initial optimal production plan, a proper component of sequence (10), and a proper plan proportional to ζ. Hence the adjustment process defined in Theorem 3 also models a way of achieving equilibrium on a disequilibrated market of "similar" commodities.

Adjustment process defined in the proof of Theorem 3 requires involvement of producers from set B_0. There are many numbers satisfying condition (12), if $n > 1$, then there are many possibilities of the choice of the set B_0 as well as, if $n_0 > 1$, then there is infinitely many possibilities of the choice of numbers s_k. Hence there are a lot of adjustment processes leading to equilibrium in a private ownership economy with modified production system according to recipe defined in the proof of Theorem 3.

Let us also notice that the set of vectors $\{s_k \cdot \zeta : k \in B_0\}$ used in the proof of Theorem 3 can be replaced by the set of vectors $\{\zeta^k : \ k \in B_0\}$ satisfying:

$$\forall k \in B_0 : \ p \circ \zeta^k = 0, \sum_{k \in B_0} \zeta^k = \zeta, \ \exists k \in B_0 : \ \zeta^k \neq 0.$$

In the adjustment process defined in the proof of Theorem 3, changes at time t, in activities of producers from set B_0 are determined by a distribution of numbers s_k which can be different for different producers. In the adjustment process defined in the proof of Theorem 1, the producers from set B_0 changes his activities in the same way, determined by vector h. The recipe defined in the proof of Theorem 1 can be applied, if producers have to eliminate a commodity from their plans of action and the production sets are modified to the linear sets (see Lipieta, 2015a). As above, the choice of set B_0, the number of steps as well as the arrangement of the rules of the adjustment process should be done before the beginning of the process. It can be done by a manager of the production sector.
At the end the following is proposed:

Theorem 4. *There is equilibrium in a transformation $E_p(2)$ of economy E_p, in which the production sector is modified through an adjustment process of the form* (5) *consisted of* 1 *step as well as every producer from set B_0 changes his productive activities in the similar way.*

Proof. Let us notice that, by (11)

$$y^* + \zeta + \omega \in X^{a_1} + \cdots + X^{a_m}.$$

Assume firstly that condition (13) is satisfied for a subset $B_0 \subset B, \ B_0 \neq \emptyset$.
As in the proof in Theorem 1, there exists a vector $h \in \mathbb{R}^\ell \backslash \{0\}$ satisfying (14). Put

$$Y^b(2) = Y^b\,(t) \cup \left\{ y^b(1) - (h \circ y^b(1)) \cdot \zeta : \ y^b(1) \in Y^b(1) \right\} \text{ for } b \in B_0,$$

$$Y^b\,(2) = Y^b(1) \text{ for } b \in B \backslash B_0,$$

$$X^a\,(2) = X^a(1), \preccurlyeq_2^a = \preccurlyeq_1^a, \ \omega^a\,(2) = \omega^a(1), \text{ for every } \ a \in A,$$

$$\widetilde{\theta}_2 = \widetilde{\theta}_1.$$

For every $k \in K$ environment $e^k(1)$ is of the form (6), message $m^k(1)$ is of the form (7) as well as $M = M^{k_1} \times \ M^{k_2} \times \cdots \times M^{k_\mathrm{K}}$.
We define mappings $f_1^k : M(1) \times E(1) \to M^k(2)$ by the formula:

$$f_1^k\,(m(1), e(1)) = (p, \ y^k(1) - \left(h \circ y^k\,(1)\right) \cdot \zeta, \ x^k(2)) \text{ for } k \in B_0,$$

$$f_1^k\,(m(1), e(1)) = (p, \ y^k(1), \ x^k(2) \text{ for } k \in K \backslash B_0.$$

where $y^k(1) = 0$ for $k \notin B$, $x^k(2) = x^{k*}$ for $k \in A$ and $x^k(2) = 0$ for $k \notin A$.
Since Walras Law is satisfied, then, for every $k \in B_0$,

$$p \circ \left(y^k(1) - \left(h \circ y^k(1) \right) \cdot \zeta \right) = p \circ y^k(1).$$

By the above, for $k \in B_0$,

$$y^{k*}(1) - (h \circ y^{k*}(1)) \cdot \zeta \in \eta_2^k(p)$$

Hence producers' maximal profits, budget sets and consequently optimal consumers' plans in economies $\mathrm{E}_p(1)$, $\mathrm{E}_q(2)$ are the same. Let us notice that

$$\sum_{k \in K} x^{k*}(2) - \sum_{k \in K} y^{k*}(2) - \omega = 0,$$

which means that sequence

$$(x^{a_1*}, \ldots, x^{a_m*}, \tilde{y}^{b_1*}, \ldots, \tilde{y}^{b_n*}) \in \left(\mathbb{R}^\ell \right)^{m+n},$$

where

$$\tilde{y}^{b*} = \begin{cases} y^{b*} - (h \circ y^{b*}) \cdot \zeta & \text{for} \quad b \in B_0 \\ y^{b*} & \text{for} \quad b \in B \backslash B_0, \end{cases}$$

is feasible in economy $\mathrm{E}_p(2)$. Hence economy $\mathrm{E}_p(2)$ is in equilibrium.
If condition (13) is not satisfied then, the reasoning goes in the same way as the proof of Theorem 2 assuming that $\tau = 2$.
The rest of the proof goes in the same way as in its first part. □

Let us notice that if condition (18) is satisfied, then the producers' response functions at time 2 can be also given by:

$$f_1^k(m(1), e(1)) = (p, \ y^k(1), \ x^k(2)) \text{ for } k \in B \backslash B_0,$$

$$f_1^k(m(1), e(1)) = (p, \ (1+s) \cdot y^k(1), \ x^k(2)) \text{ for } k \in B_0,$$

while the production sets are of the form:

$$Y^k(2) = Y^k(1) \cup \left\{ (1+s) \cdot y^k(1) : \ y^k(1) \in Y^k(1) \right\} \text{ for } k \in B \backslash B_0,$$

$$Y^k(2) = Y^k(1) \quad \text{for } k \in B_0.$$

The adjustment processes defined in Theorem 4 lead to equilibrium in economy $\mathrm{E}_p(2)$ after 1 step, under the assumption that Walras Law is satisfied at given prices. However, the changes in activities of some producers within those processes are greater in a given metric than those considered in the proofs of Theorem 1 or Theorem 2.
The 1-step adjustment processes could be applied, if the producers from the set B_0

could change their productive feasibilities to the set $Y^k(2)$ $(k \in B_0)$ defined in the proof of Theorem 4. Due to the same reasons as above if condition (13) is satisfied and $\ell > 2$, then there exist infinitely many adjustment processes and infinitely many transformations of the initial economy $\mathrm{E}_p(1)$ to economy $\mathrm{E}_p(2)$ in which equilibrium exists. Generally, to have equilibrium in economy $\mathrm{E}_q(2)$ satisfied, the producers should follow the same adjustment process. Hence producers' activities should be coordinated by a manager especially that set B_0 does not have to be unique. As earlier, the adjustment process defined in the proof of Theorem 2 requiring involvement of producers from set B_0.

Let us emphasise that the assumptions considered in Theorems 1-4 are not equivalent to those used by Arrow and Debreu (Arrow, Debreu 1954, p. 266). Hence, in some cases, Theorems 1-4 can be regarded as next, alternative versions of the classical theorem proved by Arrow and Debreu (1954) on the existence of equilibrium in an exchange economy with production. Similarly, the adjustment processes defined in the proofs of Theorems 1-4 differ in the definition of the response functions from the adjustment processes considered so far (see, for example Arrow, Hurwicz, 1958, 1959; Hurwicz, 1987), which concentrate, above all, on determining equilibrium prices in the models under study. However, as it was shown in Example 1, in some models of the economy, equilibrium cannot be obtained at given prices. In such cases one of the procedures defined in the proofs of Theorems 1-4 could be employed, especially that their use does not require strong mathematical assumptions (such as differentiability or quasi-concavity of utility functions) which were usually assumed in classical theorems concerning the properties of equilibrium in the competitive economy.

At the end, let us notice that during the adjustment processes defined in the proofs of Theorems 1-4 innovations in Schumpeter's meaning appear in the market at time $t \in \{2, \dots, \tau\}$, if

$$Y^{b_0}(t) \not\subset Y^{b_0}(t-1)\,, \text{ for } t \in \{1, \dots, \tau\} \text{ and } b_0 \in B_0, \tag{20}$$

as well as

$$Y^{b_0}(t) \not\subset Y^{b}\left(\tilde{t}\right), \text{ for every } b \in B \text{ and } \tilde{t} = 1, \dots, t-1. \tag{21}$$

If conditions (20) and (21) are satisfied, then some changes introduced by producer b_0 into his activity, in the framework of a given adjustment process, are interpreted as production innovations (see Schumpeter, 1912; Lipieta, 2013). During such adjustment process the economy moves in the direction of equilibrium, no new products appear but new technologies at least of one producer can be distinguished. So, during a process of setting of equilibrium, an innovation could or could not appear on the market. In both above cases, such process can be regarded as one of the stages of Schumpeterian economic evolution.

5 Conclusions

In real economies, an adjustment process should be chosen due to various conditions, such as particular initial conditions, technological possibilities and the aims of firms or aims of the central management. Under natural assumptions, it is feasible to drive the changes in the producers' activities to reach the equilibrium in the Debreu economy, although the application of some of the determined adjustment processes into real markets may be a real challenge to achieve.

If, for every $t = 1, \ldots, \tau$ and $b \in B$, $Y^b(t) \subset Y^b(t-1)$, then the adjustment processes defined in the proofs of Theorems 1-4 can be interpreted as adjustment processes in the same economy. However, above all, the defined adjustment processes could be interpreted as the various ways to reach equilibrium in the Debreu economy in which the production system can be modified.

Under the perfect rationality assumption, every producer can determine his optimal adjustment process with respect to a given criterion. Solving the problem of existence, uniqueness and specification of an adjustment process preferred by all producers under given initial conditions could simplify the problem of the choice and applying an adjustment procedure, because in such a situation every economic agent would change his activity in the same way without any additional incentives. The above still remains under our consideration.

Acknowledgments

I would like to thank Jakub Growiec and the anonymous Referee who offered very helpful comments, which allowed me to improve the quality of the paper.

This work is supported by National Science Centre in Poland, GRANT 2014/13/B/HS4/03348.

References

[1] Adamowicz Z., Zbierski P., (1997), *Logic of Mathematics: A Modern Course of Classical Logic*, Wiley, New York.

[2] Andersen E. S., (2009), *Schumpeter's Evolutionary Economics*, Anthem Press, London.

[3] Arrow K. J ., Debreu G., (1954), Existence of an equilibrium for a competitive economy, *Econometrica*, 22, 265–290.

[4] Arrow K. J., Hurwicz L., (1958), On the stability of the competitive equilibrium, I, *Econometrica*, 26, 522–552.

[5] Arrow K. J., Hurwicz L., (1959), On the stability of the competitive equilibrium, II, *Econometrica*, 27, 82–109.

[6] Debreu G., (1959), *Theory of value*, Wiley, New York.

[7] Hodgson G. M., (1993), *Economics and Evolution: Bringing Life Back into Economics*, Polity Press, Cambridge.

[8] Hurwicz L., (1987), Incentive Aspects of Decentralization, [in:] *Handbook of Mathematical Economics*, vol. 3, Arrow K. J., Intriligator M. D. [eds.], Elsevier, Amsterdam.

[9] Hurwicz L. (1994), Economic design, adjustment processes, mechanisms, and institutions, *Review of Economic Design*, 1(1), 1–14.

[10] Hurwicz L., Reiter S., (2006), *Designing Economic Mechanism*, Cambridge University Press, New York.

[11] Joyce P., (1984), The Walrasian Tâtonnement Mechanism and Information, *RAND Journal of Economics*, 15(3), 416–425.

[12] Koopmans T., C. [ed.], (1951), *Activity Analysis of Production and Allocation*, John Wiley and Sons, New York.

[13] Lipieta A., (2013), Mechanisms of Schumpeterian Evolution, [in.:] *Innovative economy as the Object of Investigation in Theoretical Economics*, Malawski A. [ed.], Cracow, University of Economics Press, Cracow, 94–119.

[14] Lipieta A., (2015a), Producers' Adjustment Trajectories Resulting in Equilibrium in the Economy with Linear Consumption Sets, *Central European Journal of Economic Modelling and Econometrics*, 7, 187–204.

[15] Lipieta A., (2015b), Existence and Uniqueness of the Producers' Optimal Adjustment Trajectory in the Debreu-type economy, *Mathematical Economics* 11(18), Wrocław University of Economics Press ISSN:1773 9707, 55–68.

[16] Lipieta A., Malawski A., (2016), Price versus Quality Competition: In Search for Schumpeterian Evolution Mechanisms, *Journal of Evolutionary Economics*, 26(5), doi:10.1007/s00191-016-0470-8, 1137–1171.

[17] Lipieta A. (2016), Adjustment processes in the Debreu-type economy. Matematyka i informatyka na usługach ekonomii, [in.:] *Wybrane współczesne problemy wzrostu gospodarczego informatyki ekonomicznej*, Jurek W. [ed.], Poznań University of Economics Press, 75–86.

[18] Maćkowiak P., (2010), The existence of equilibrium without fixed-point arguments, *Journal of Mathematical Economics*, 46, 1194–1199.

[19] Marschak T., (1972), Computation in organization: The comparison of price mechanisms and other adjustment processes [in:] *Decision and Organisation*, McGuire C. B., Radner R. [eds.], Amsterdam.

[20] Marschak T., (1987), Organization Design [in:] *Handbook of Mathematical Economics*, vol. 3, Arrow, K. J., Intriligator, M. D. [eds.], Elsevier, Amsterdam.

[21] Mas-Colell A., Whinston M. D., Green J. R., (1995), *Microeconomic Theory*, Oxford University Press, New York.

[22] Moore J., (2007), *General Equilibrium and Welfare Economics*, Springer, Berlin-Heidelberg-New York.

[23] Radner R., (1972), Existence of Equilibrium of Plans, Prices and Price Expectations in a Sequence of Markets, *Econometrica*, 40(2), 289–303.

[24] Schumpeter J., (1912), *Die Theorie der wirtschaftlichen Entwicklung*, Leipzig: Duncker & Humblot. English translations: *The theory of economic development*, Cambridge, MA: Harvard University Press 1934 and A Galaxy Book, New York, Oxford University Press 1961.

[25] Schumpeter J. A., (1950), *Capitalism, socialism and democracy*, Harper, New York.

[26] Shionoya Y., (2015), Schumpeter and evolution: an ontological exploration. `http://www.lib.hit-u.ac.jp/service/tenji/amjas/Shionoya18.pdf`, Accessed 29 Dec 2015.

[27] Uzawa H., (1960), Walras' Tâtonnement in the Theory of Exchange, *Review of Economic Studies*, 27(3), 182–194.

[28] Walras L., (1954), *Elements of Pure Economics*, Allen and Unwin, London.